PERIODIC DISLOCATION MOTION IN A RELAXING MEDIUM

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We consider periodic helical dislocation motion in a para-elastic medium under variable external stresses. The para-elastic properties of the medium are determined by the short-range-order parameters between atoms of the different components of the alloy. The solution of the nonlinear dislocation equation of motion is obtained in four different regions of the amplitude-frequency space. The conditions are indicated under which dislocation motion is viscous and is in the nature of breakaway from the polarization atmosphere.

1. In the displacements of undivided helical dislocation in an isotropic elastic medium, internal friction occurs in connection with para-elastic relaxation [1]. The dislocation motion is defined by the equation

$$\mathrm{f} + \mathrm{f}^{(e)} + \mathrm{f}^{(a)} = 0 \qquad (f_i^{(e)} = -\varkappa\xi_i, \quad f_i = \varepsilon_{ijk} s_j b_l \sigma_{kl}) \tag{1.1}$$

Here f_i are the forces from the side of the external stresses σ_{kl} acting on a straight line dislocation of unit length along the direction s_j with Burgers vector b_l; the $f_i^{(e)}$ are the linear tensile strengths of a dislocation segment of length L defined by the displacement ξ_i from the equilibrium position in the absence of external stresses. The quasielastic coefficient $\varkappa$ is defined by the equation

$$\varkappa = \frac{\mu b^2}{4\pi L^2} \ln \frac{L}{r_0} \tag{1.2}$$

in which μ is the shear modulus and $r_0 \sim b$ is the radius of the dislocation kernel. The interaction between the dislocation and the polarization atmosphere $f_i^{(a)}$ is determined from the equation for the energy of that interaction

$$U^{(a)} = \int \sigma_{ik}^{(a)} u_{ik}^{(d)} dS \tag{1.3}$$

where $\sigma_{ik}^{(a)}$ is the stress field created by the atmosphere; $u_{ik}^{(d)}$ is the dislocation deformation. Integration in (1.3) is over a plane perpendicular to the line of dislocation excluding a region of radius r_0 of the dislocation kernel. In the case of helical dislocation along the third axis, the nonzero components of $u_{ik}^{(d)}$ are

$$u_{13}^{(d)} = -\frac{b}{4\pi} \frac{y}{x^2 + y^2}, \qquad u_{23}^{(d)} = \frac{b}{4\pi} \frac{x}{x^2 + y^2} \tag{1.4}$$

If we consider a crystal with polarization atmospheres about the dislocation as a para-elastic medium, the stress field created by the atmosphere in the approximation for a standard linear body can be written

$$\sigma_{ik}^{(a)} = \Delta\lambda_{iklm} \int_0^\infty e^{-t'/\tau} u_{lm}^{(d)\cdot} (t - t')\, dt' \tag{1.5}$$

Voronezh. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 79-85, November-December, 1970. Original article submitted March 15, 1970.

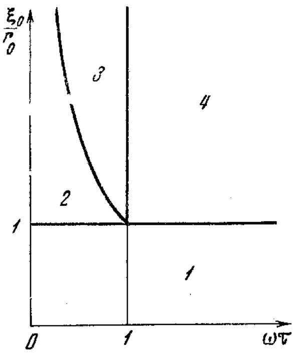

Fig. 1

Here $\Delta\lambda_{iklm}$ is the tensor defect of the modules of the medium, and τ is the relaxation time. For example, for alloys with replacement on a body-centered cubic lattice basis, the expansion for $\Delta\lambda_{iklm}$ in the case of short-range-order relaxation has the following form:

$$\Delta\lambda_{iklm} = \Delta\mu\left[\delta_{ik}\delta_{lm} + \delta_{il}\delta_{km} + \delta_{im}\delta_{kl} - 2\sum_{j=1}^{3}\delta_{ij}\delta_{kj}\delta_{lj}\delta_{mj}\right] \tag{1.6}$$

The quantity $\Delta\mu$ is defined by the parameters of the interatomic interaction in the alloy.

Let $\xi(t)$ denote the dislocation displacement along the x axis under the action of external forces. Then, using (1.4) and (1.5) and integrating in (1.3) with respect to the spatial coordinates, we obtain an expression for the energy of the interaction between the dislocation and the atmosphere which depends on the previous history of the dislocation motion:

$$U^{(a)} = \frac{\Delta\mu b^2}{6\pi}\int_0^\infty e^{-t'/\tau}\frac{\Delta\xi\Delta\xi^{\cdot}}{\Delta\xi^2 + r_0^2}dt' \qquad (\Delta\xi = \xi(t) - \xi(t - t')) \tag{1.7}$$

where $\Delta\xi^{\cdot}$ is the derivative of $\Delta\xi$ with respect to t'. If we substitute in (1.1) explicit expressions for the forces, we obtain an integral equation for the dislocation motion:

$$\beta\int_0^\infty e^{-t'/\tau}\frac{r_0^2 - \Delta\xi^2}{(\Delta\xi^2 + r_0^2)^2}\Delta\xi^{\cdot}dt' + \varkappa\xi = \sigma b \qquad (\beta = \Delta\mu b^2/6\pi) \tag{1.8}$$

In what follows we consider periodic solutions of Eq. (1.8) for the case of harmonic external stresses $\sigma = \sigma_0 \sin \omega t$.

Figure 1 shows the region of the amplitude-frequency space inside which we shall find below various periodic dislocation motions and obtain expressions for the internal friction, using simplifications of Eq. (1.8).

2. For small dislocation displacements $\xi \ll r_0$, instead of (1.8) we obtain a linear integral equation:

$$\frac{-\beta}{r_0^2}\int_0^\infty e^{-t'/\tau}\xi^{\cdot}dt' + \varkappa\xi = \sigma b \tag{2.1}$$

Thus, in this case relaxation is independent of the amplitude of the internal friction:

$$Q^{-1} = \frac{NLb^2\mu\beta}{2\varkappa^2 r_0^2(1 + \beta/\varkappa r_0^2)}\,\frac{\omega\tau_1}{1 + \omega^2\tau_1^2} \tag{2.2}$$

Here N is the dislocation number in unit volume, and the relaxation time is

$$\tau_1 = \tau(1 + \beta/\varkappa r_0^2) \tag{2.3}$$

Equations (2.2) and (2.3) show that the height of the relaxation peak and the relaxation time depend on the degree of binding of the dislocation atmosphere. For weakly bound atmospheres ($\beta/\varkappa\, r_0^2 \ll 1$) the relaxation time is $\tau_1 \approx \tau$, and the height of the peak increases as the degree of binding of the dislocation atmosphere increases. In the case of strong binding ($\beta/\varkappa r_0^2 \gg 1$) the height of the peak is determined only by the density of the moving dislocations, and the relaxation time τ_1 can be much greater than the relaxation time τ of the medium.

3. We consider the case of oscillations of the dislocation segment with large amplitude $\xi_0 \gg r_0$ and low frequency $\omega \ll r_0/\tau\xi_0$ (region 2 in Fig. 1). In these conditions the polarization cloud moves with the dislocations, which corresponds to visco-elastic dislocation motion. Equation (1.8) takes the form

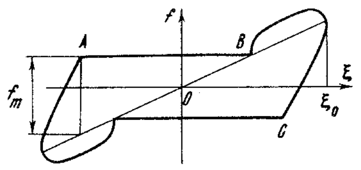

Fig. 2

$$\varkappa\xi + \eta\xi^{\cdot} = \sigma b \quad (\eta = \beta\tau / r_0^2) \tag{3.1}$$

where η is the viscosity. Internal friction is relaxational in nature:

$$Q^{-1} = \frac{NLb^2\mu}{2\varkappa}\,\frac{\omega\tau_2}{1+\omega^2\tau_2^2} \qquad \left(\tau_2 = \frac{\beta\tau}{\varkappa r_0^2}\right) \tag{3.2}$$

Here τ_2 is the relaxation time.

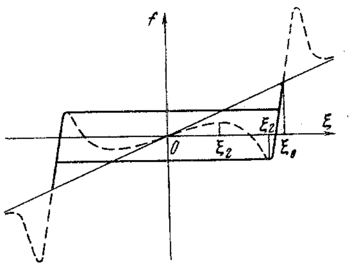

Fig. 3

From the condition $\omega \ll r_0/\tau\xi_0$ and the condition that the relaxation maximum is reached, $\omega\tau_2 = 1$, it follows that the relaxation maximum (3.2) can be reached when the atmosphere is strongly bound $(\varkappa\xi_0 \ll \beta/r_0)$.

We note that analysis of the experimental results on Zener relaxation has to take into account the dislocation contribution to internal friction. In particular, this is because internal friction is dependent on the orientation in multicrystalline solid replacement solutions, since the dependences on orientation of the Zener and dislocation relaxation are significantly different.

4. We consider the motion of the dislocation segment with frequency $\omega < 1/\tau$ and $r_0 \ll \xi_0$ (regions 2, 3 in Fig. 1). To a good approximation we can assume that the force on the dislocation moving with instantaneous velocity $\xi^{\cdot}(t)$ coincides with the force on a dislocation moving uniformly with the same velocity. If we substitute $\Delta\xi = \xi^{\cdot} t'$ in Eq. (1.8) and integrate, we have

$$f^{(a)}(\xi^{\cdot}) = -\beta / \tau\xi^{\cdot}\,[\mathrm{ci}\,(r_0/\tau\xi^{\cdot})\,\cos\,(r_0/\tau\xi^{\cdot}) + \mathrm{si}\,(r_0/\tau\xi^{\cdot})\,\sin\,(r_0/\tau\xi^{\cdot})] \tag{4.1}$$

The curve $f^{(a)}(\xi^{\cdot})$ leaves the origin and has a maximum $\approx\beta/5r_0$ for $\xi^{\cdot}_m \approx 4r_0/\tau$.

Investigation of the asymptotic behavior of (4.1) shows that

$$f^{(a)} \sim \xi^{\cdot} \quad \text{for} \quad \xi^{\cdot} \ll r_0/\tau, \quad f^{(a)} \sim \ln\xi^{\cdot}/\xi^{\cdot} \quad \text{for} \quad \xi^{\cdot} \gg r_0/\tau$$

which agrees with the results of [2]. Below it is convenient to use the equation for the force $f^{(a)}(\xi^{\cdot})$ obtained in [2] because of its simplicity compared with (4.1).

The dislocation equation of motion has the form

$$\sigma b = \varkappa\xi + \frac{\beta}{\xi^{\cdot}\tau}\ln\left|1 + \left(\frac{\xi^{\cdot}\tau}{r_0}\right)^2\right| \tag{4.2}$$

We have not succeeded in solving this equation exactly. Hence we first consider the case of weak interactions between the dislocation and the atmosphere. In the linear approximation we obtain the following solution of Eq. (4.2) in terms of the small nondimensional parameter $\beta/\varkappa\xi_0^2 \ll 1$:

$$\xi(t) = \xi_0 \sin\omega t - \frac{\beta}{\varkappa\xi_0\omega\tau\cos\omega t}\ln\left|1 + \left(\frac{\omega\tau\xi_0}{r_0}\right)^2\cos^2\omega t\right| \tag{4.3}$$

From (4.3) we determine the internal friction:

$$Q^{-1} = \frac{8\pi NL\beta}{\varkappa\xi_0^2\omega\tau}\ln\left|\frac{1}{2}\left(1 + \left(\frac{\omega\tau\xi_0}{r_0}\right)^2\right)^{1/2}\right| \tag{4.4}$$

Equation (4.4) for the internal friction as a function of the frequency has a maximum at which the frequency is

$$\omega_m = r_0 z_1 / \tau\xi_0 \tag{4.5}$$

and it diminishes as the amplitude of the oscillations increases. In (4.5) the dimensionless coefficient $z_1 \sim 1$ is the nonzero solution of the equation

$$2z - (1 + z + \sqrt{1+z})\ln|{}^1/_2(1 + \sqrt{1+z})| = 0 \tag{4.6}$$

The maximum value of the internal friction

$$Q_m^{-1} = \frac{8\pi NL\beta}{\varkappa r_0 \xi_0} \ln \frac{1+\sqrt{2}}{2} \tag{4.7}$$

decreases as the amplitude of the oscillations increases and depends weakly on the temperature of the specimen.

We note that the above results agree qualitatively with the numerical computations of [3].

In the case when the atmosphere is strongly bound to the dislocation ($\beta/\varkappa\xi_0^2 > 1$), the para-elastic behavior of the medium strongly affects the motion of the dislocation segment. For sufficiently large amplitude σ_0 and frequency ω periodic motion of the dislocation segment occurs with breakaway from the moving atmosphere of the elastic polarization of the medium. To compute the internal friction and the conditions under which such a phenomenon can arise, we use a simple approximation for the relation between the force and the velocity:

$$f^{(a)}(\xi^{\cdot}) = \begin{cases} \eta\xi^{\cdot}, & |\xi^{\cdot}| \leqslant r_0/5\tau = \xi_m^{\cdot} \\ 0, & |\xi^{\cdot}| > \xi_m^{\cdot} \end{cases} \tag{4.8}$$

The viscosity η, as in (3.1), is defined by the equation $\eta = \beta\tau/r_0^2$. The boundary value of the velocity $\xi_m^{\cdot}$, defining the interval in which $f^{(a)}(\xi^{\cdot})$ is linear, is found from the condition that the force is given by (4.8), the velocity at the maximum force being defined by (4.1).

The moment of breakaway of the dislocation from the atmosphere is determined from the condition that the force on the dislocation from the side of the external stresses and tension exceeds the maximum binding force of the atmosphere; we have

$$\sigma b = \varkappa\xi + f_m \qquad (f_m = \beta / 5r_0) \tag{4.9}$$

To determine the moment of breakaway t_0 we consider the motion of dislocation in accordance with Fig. 2, which shows the relation between the displacement ξ and the force f on the dislocation when there is a single breakaway from the moving atmosphere. At time t_0 the dislocation breaks away from the point A and reaches a position defined by the quasielastic tension B. The coordinates of the point B in the $f\xi$ plane are

$$\begin{aligned} f_B &= \sigma_0 b \sin \omega t_0, \\ \xi_B &= \sigma_0 b \varkappa^{-1} \sin \omega t_0 \end{aligned} \tag{4.10}$$

After time τ the dislocation moving near the point B surrounds the polarization atmosphere, and the subsequent motion of the dislocation is again limited by the relaxation properties of the polarization atmosphere and is subject to condition (4.2).

Ignoring an interval of time $\sim\tau$, which is small in comparison with the time for the dislocation to move to the next breakaway, we find an equation for $\xi(t)$ from the solution of (1.1) with the approximation (4.8):

$$\xi(t) = Ce^{-t/\tau_2} + \frac{\xi_0}{1+\omega^2\tau_2^2}(\sin\omega t - \omega\tau_2\cos\omega t) \tag{4.11}$$

where C is determined from the initial condition (4.10):

$$Ce^{-t_0/\tau_2} = \frac{\xi_0\omega\tau_2}{1+\omega^2\tau_2^2}(\cos\omega t_0 + \omega\tau_2 \sin\omega t_0) \tag{4.12}$$

It is easy to verify from (4.11) and (4.12) that $\xi^{\cdot}(t_0) = 0$. This corresponds to the fact that after displacement into a new position the dislocation begins to move again with zero velocity and the tangent to the curve BC at B is parallel to the f axis. If we equate the force from the side of the external stresses and the linear tension on the dislocation at the maximum force f_m, we obtain the condition for breakaway at time $t_0 + \pi/\omega$ in the following form:

$$\xi_0 \sin \omega t_0 + C \exp\left(-\frac{t_0}{\tau_2} - \frac{\pi}{\omega\tau_2}\right) = \frac{\xi_0}{1+\omega^2\tau_2^2}(\sin\omega t_0 - \omega\tau_2 \cos\omega t_0) + \xi_m \tag{4.13}$$

$$(\xi_m = f_m / \varkappa)$$

The solution of Eq. (4.13) is

$$t_0 = 2\omega^{-1} \operatorname{arc\,tg} \theta$$

$$\theta = \left(1 + \frac{\xi_m \nu}{\xi_0 \omega \tau_2 \chi}\right)^{-1} \left\{\omega\tau_2 + \left[\nu\left(1 - \frac{\xi_m^2 \nu}{\xi_0^2 \omega^2 \tau_2^2 \chi^2}\right)\right]^{1/2}\right\} \tag{4.14}$$

$$(\nu = 1 + \omega^2\tau_2^2, \quad \chi = 1 + e^{-\pi/\omega\tau_2})$$

From the condition that the expression under the square root sign is positive, we find the lower boundary for the amplitude of the external stresses σ_* for which such oscillations of the dislocation segment are possible:

$$\sigma_* = \frac{\sigma_m \nu^{1/2}}{\omega\tau_2\chi}, \qquad \sigma_m = f_m / b \tag{4.15}$$

The quantity σ_* is a function of the frequency. At low frequencies $\omega\tau_2 \ll 1$ the amplitude is $\sigma_* \sim \sigma_m/\omega\tau_2$, while in the case of high frequencies not exceeding $1/\tau_2$ it is $\sigma_* > \sigma_m/2$.

The upper boundary of the stresses under which motion with a single breakaway (in a half-period) of the dislocation from the atmosphere occurs is determined from the condition that the sum of the elastic and tension forces only exceeds f_m once in a half-period. Otherwise, as the stress increases, the dislocation breaks away a second time from the atmosphere. The internal friction is defined as

$$Q^{-1} = \frac{1NLb^2\mu}{\varkappa}\left[\frac{\pi\omega\tau_2}{\nu} + \frac{2\xi_m^2}{\xi_0^2} - \frac{2\xi_m}{\xi_0\nu}(\sin\omega t_0 + \omega\tau_2\cos\omega t_0)\right] \tag{4.16}$$

If the external stresses $\sigma_0 < \sigma_*$, then

$$Q^{-1} = \frac{2\pi NLb^2\mu\omega\tau_2}{\varkappa\nu} \tag{4.17}$$

As σ_0 passes through the point σ_*, the internal friction changes discontinuously by an amount

$$\Delta Q^{-1} = \frac{4NLb^2\mu\omega^2\tau_2^2\chi^2}{\varkappa\nu}(1 - 2/\nu\chi) \tag{4.18}$$

We see from this that if $\omega < 1/\tau_2$, then $\Delta Q^{-1} < 0$, while for $\omega > 1/\tau_2$ the discontinuity is positive, and as the frequency increases, $\Delta Q^{-1} \rightarrow 16\, N\, Lb^2 \mu\varkappa^{-1}$. Thus, for sufficiently large frequencies the internal friction is determined by the force of breakaway from the moving polarization atmosphere.

5. We consider high-frequency dislocation oscillations in region 4 of Fig. 1. If we expand the exponential function in the integrand of (1.8) in a series of powers of $1/\omega\tau$ and retain only two terms of the expansion, we obtain

$$\sigma b = \varkappa\xi - \frac{\beta}{2\pi}\int_0^{2\pi} \frac{\Delta\xi\, dx'}{\Delta\xi^2 + r_0^2} + \frac{\beta}{2\pi\omega\tau}\int_0^{2\pi} \frac{\omega\xi x'\, dx'}{\Delta\xi^2 + r_0^2} \tag{5.1}$$

$$(x' = \omega t)$$

The second term in (5.1) is the force on the dislocation segment from the side of the stationary polarization cloud established as a result of the periodic motion of the dislocation. The last term, which is much less than the second, is the viscous force on the moving dislocation.

In the case of a weak atmosphere the viscous component plays a fundamental role in the internal friction. If we consider it as a small perturbation, we can determine the addition to the zero-order approximation:

$$\xi_1(t) = \frac{\beta\xi_0}{2\pi\omega\tau\varkappa}\int_0^{2\pi} \frac{[\sin x - \sin(x - x')]\, x'\, dx'}{\xi_0^2[\sin x - \sin(x - x')]^2 + r_0^2} \tag{5.2}$$

We have ignored the second term in (5.1) since in the approximation it makes no contribution to the dislocation energy. Using (5.2), we find the oscillation energy absorbed in a period and the internal friction:

$$Q^{-1} = \frac{8\pi NLb^2\mu\beta}{\varkappa^2\xi_0^2\omega\tau}\ln\frac{\xi_0}{r_0} \tag{5.3}$$

If the binding between the dislocation and the atmosphere is important, the standard distribution of the polarization atmosphere has a significant effect on the nature of the dislocation motion; since the dislocation for a relatively large part of the time moves near the extreme positions, the density of the atmosphere there is maximal. The situation may be complicated when the periodic dislocation motion in the potential field of a distributed atmosphere can breakaway from the extreme positions where the force maintaining the dislocation of the polarization atmosphere is maximal.

We consider the particular case of motion in which the dislocation breaking away from the cloud at one of the extreme positions rapidly moves to the opposite extreme. The conditions for the existence of such dislocation oscillations are found approximately assuming that the polarization atmosphere is formed by the moving dislocation, which for the basic part of its time is found in the neighborhood of the extreme positions. Then Eq. (5.1) can be written

$$\sigma b = \varkappa\xi + \frac{\beta}{2}\left[\frac{\xi_0+\xi}{(\xi_0+\xi)^2+r_0^2} - \frac{\xi_0-\xi}{(\xi_0-\xi)^2+r_0^2}\right] \tag{5.4}$$

The contour of the force field acting on the dislocation and the hysteresis loop formed because the force f depends on the displacement ξ of the dislocation for high-frequency oscillations are shown in Fig. 3. The level of the external stresses at which dislocation motion with breakaway is possible is determined by the relation between the extreme values f_1 and f_2 of $f(\xi)$:

$$f_2 \leqslant f_1 \leqslant \sigma_0 b \tag{5.5}$$

Noting that $\xi_1 \approx \xi_0 - r_0$, $\xi_2 \approx \xi_0 - (\beta/2\varkappa)^{1/2}$, after computing f_1 and f_2,(5.5) can be put in the form

$$\beta / 8b^2 \leqslant \sigma_0 \leqslant \beta / 8b^2 + (\beta\varkappa / 2b^2)^{1/2} \tag{5.6}$$

For stresses σ_0 less than $\sigma_1 = \beta/8b^2$ the hysteresis loop does not occur. When the level σ_1 is exceeded, all the dislocation loops, except the smallest, which collapse under stresses $\sim\sigma_1$ at a distance $\sim r_0$, move with breakaway at the points $\pm\xi_1$. If then σ_0 increases, the longest dislocation segments cease to move with a breakaway since for them the polarization atmosphere becomes more homogeneous and the force field becomes smoother. Hence the internal friction must decrease as σ_0 increases, the rate of decrease being determined by the distribution of the dislocation segments in length.

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